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Book Reviews

B. CARL AND I. STEPHANI, *Entropy, Compactness and the Approximation of Operators*, Cambridge University Press, 1990, 277 pp.

The subject matter of this tract is at one of the interfaces of functional analysis (operator theory) and approximation theory. It is concerned with the properties and relationship between entropy and approximation quantities, as applied to linear operators. Entropy quantities in their simplest form are measures of coverings, and as such may be considered as measures of the degree of compactness of an operator. The approximation quantities considered are the approximation, Kolmogorov, and Gel'fand numbers (generally called n-widths in the approximation theory literature), which measure the degree of approximability of linear operators by finite rank operations. Chapter 3 discusses the relationship between the entropy and the approximation quantities. These are given by "Bernstein and Jackson type" inequalities. In Chapter 4 the influence of these quantities on the spectral properties, i.e., distribution of eigenvalues, is studied. Chapter 5 is devoted to operators with values in a Banach space $C(X)$ of continuous functions on a compact metric space X. The modulus of continuity of such operators is used to relate the entropy and approximation numbers. This is a very well-written book which would make an excellent seminar text.

ALLAN PINKUS

G. GASPER AND M. RAHMAN, *Basic Hypergeometric Series*, Cambridge University Press, 1990, 287 pp.

Hypergeometric functions form a very large class of special functions which are relevant for various applications in differential equations, power series expansions, and (classical) orthogonal polynomials. These are series for which the ratio of two consecutive terms c_n and c_{n+1} is a rational function of n. For basic hypergeometric functions this ratio is a rational function of q^n for some complex parameter q. The limiting case $q \to 1$ usually gives hypergeometric functions, but quite often there are several q-extensions of a hypergeometric function. The book begins with classical work by Heine, Jackson, and Bailey and the q-extensions of the binomial theorem, Gauss' summation formula for the evaluation of a ${}_2F_1$ at the point 1, and the gamma and beta function. The next two chapters contain a wealth of formulas starting from a very high level with many parameters (up to 20 in some cases) and then taking limits or special values down to simpler formulas. Understanding the material requires advanced calculus, complex analysis, and a good dose of courage to work through the computations. Be careful though; the q-disease is contagious! The last three chapters basically deal with orthogonal polynomials, starting from the Askey–Wilson q-extension of the beta integral. The "classical" orthogonal polynomials (orthogonal polynomials that are basic hypergeometric functions) and their orthogonality are discussed and applications are given, including product formulas for the Askey–Wilson polynomials, q-analogues of Clausen's formula (important for proving positivity), and elements from the theory of partitions of positive integers.

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